Global vs. Zonal Approaches in Hybrid RANS-LES Turbulence Modelling

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Abstract. The paper will provide an overview of hybrid RANS-LES methods currently used in industrial flow simulations and will evaluate the models for a variety of flow topologies. Special attention will be devoted to the aspect of global vs. zonal approaches and aspects related to interfaces between RANS and LES zones.

1 Introduction

Historically, industrial CFD simulations are based on the Reynolds Averaged Navier-Stokes Equations (RANS). For many decades, the only alternative to RANS was Large Eddy Simulation (LES), which has however failed to provide solutions for most flows of engineering relevance due to excessive computing power requirements for wall-bounded flows. On the other hand, RANS models have shown their strength essentially for wall-bounded flows, where the calibration according to the law-of-the-wall provides a sound foundation for further refinement. For free shear flows, the performance of RANS models is much less uniform. For this reason, hybrid models are under development , where large eddies are only resolved away from walls and the wall boundary layers are entirely covered by a RANS model (e.g. Detached Eddy Simulation – DES or Scale-Adaptive Simulation – SAS). A further step is the application of a RANS model only in the innermost part of the wall boundary layer and then to switch to an LES model for the main part of the boundary layer. Such models are termed Wall Modelled LES (WMLES). Finally, for large domains, it is frequently only necessary to cover a small portion with Scale-Resolving Simulation (SRS) models, while the majority of the flow can be computed in RANS mode. In such situations,

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zonal or embedded LES methods are attractive. Such methods are typically not new models in the strict sense, but allow the combination of existing models/technologies in a flexible way in different zones of the simulation domain. Important elements of zonal models are interface conditions, which convert turbulence from RANS mode to resolved mode at pre-defined locations. In most cases, this is achieved by introducing synthetic turbulence based on the length and time scales from the RANS model.

The challenge for the engineer is to select the most appropriate model for the intended application. Unfortunately, none of the available SRS models is able to efficiently cover all industrial flows. A compromise has to be made between generality and CPU requirements. The paper will discuss the main different models available in today's industrial CFD codes and provide some guidelines as to their optimal usage.

2 Hybrid RANS-LES Turbulence Models

There is a large variety of hybrid RANS-LES models with often somewhat confusing naming conventions concerning the range of turbulence eddies they will resolve. On close inspection, many of these models are slight variations of the Detached Eddy Simulation (DES) concept of Spalart (1997, 2000) with very similar behavior. The present paper will provide a review of models which are in, or at the verge of, industrial use – which reduces the model variety considerably. Naturally, the authors will focus on the methods employed in our own CFD codes, and more specifically ANSYS-Fluent and ANSYS-CFX. For a general overview of SRS modelling concepts see e.g. Fröhlich and von Terzi (2008) or Sagaut et al. (2006).

It is not the goal of this paper to provide the full detail of all models, but to highlight the main concepts and their consequences for the industrial usage. Therefore only a schematic description of the models will be provided.

2.1 Scale-Adaptive Simulation (SAS)

SAS is a concept which enables Unsteady RANS (URANS) models to operate in SRS mode. This is achieved by the introduction of the second derivative of the velocity field into the turbulence scale equation. The derivation is based on a theory of Rotta (see e.g. Rotta, 1972), resulting in an exact equation for the turbulence length scale. This equation served as a basis for a term-by-term modelling of the length-scale equation. The details of the derivation and numerous examples can be found in Menter and Egorov (2010), Egorov et al. (2010). The essential quantity, which appears in the equations and which allows the switch to SRS mode is the von Karman length scale L_{vk}:

$$L_{vK} = \kappa \left| \frac{\bar{U}'}{\bar{U}''} \right|; \quad \bar{U}'' = \sqrt{\frac{\partial^2 \bar{U}_i}{\partial x_k^2} \frac{\partial^2 \bar{U}_i}{\partial x_j^2}}; \quad \bar{U}' = S = \sqrt{2 \cdot S_{ij} S_{ij}}; \quad S_{ij} = \frac{1}{2}\left(\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i} \right)$$

L_{vk} allows the SAS model to detect resolved unsteady structures in the simulation and to reduce the eddy-viscosity accordingly. Due to the lower eddy-viscosity, new smaller structures can be generated resulting in a turbulence cascade down to the grid limit. At the grid limit, different limiters can be employed ensuring a proper dissipation of turbulence. The advantage of SAS models is that the limiters do not affect the RANS behavior of the model.

2.2 Detached Eddy Simulation (DES)

Detached Eddy Simulation (DES) has been proposed by Spalart and co-workers (Spalart et al., 1997, 2000, Travin et al., 2000, Strelets 2001), to eliminate the main limitation of LES models, by proposing a hybrid formulation which switches between RANS and LES based on the grid resolution provided. By this formulation, the wall boundary layers are entirely covered by the RANS model and the free shear flow portions are typically computed in LES mode. The formulation is mathematically relatively simple and can be built on top of any RANS turbulence model. DES has attained significant attention in the turbulence community as it allows the inclusion of SRS capabilities into every day engineering flow simulations.

Within the DES model, the switch between RANS and LES is based on a criterion like:

$$\begin{aligned} C_{DES}\Delta > L_t \quad &\rightarrow \quad RANS \quad ; \quad \Delta_{\max} = \max\left(\Delta_x, \Delta_y, \Delta_z\right) \\ C_{DES}\Delta \le L_t \quad &\rightarrow \quad LES \end{aligned}$$

The actual formulation for a two-equation model is (e.g. k-ω):

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho \bar{U}_j k)}{\partial x_j} = P_k - \rho \frac{k^{3/2}}{\min\left(L_t; C_{DES}\Delta_{\max}\right)} + \frac{\partial}{\partial x_j}\left[(\mu + \frac{\mu_t}{\tilde{\sigma}_\kappa}) \frac{\partial k}{\partial x_j} \right]$$

$$L_t = \frac{k^{3/2}}{\varepsilon} = \frac{\sqrt{k}}{\beta^* \omega}$$

As the grid is refined below the limit $\Delta_{\max} < L_t$ the DES-limiter is activated and switches the model from RANS to LES mode. The intention of the model is to run in RANS mode for attached flow regions, and to switch to LES mode in detached regions away from walls.

It is important to note that the DES limiter can already be activated by grid refinement inside attached boundary layers. This is undesirable as it affects the RANS model by reducing the compute eddy viscosity which, in term, can lead Grid-Induced Separation (GIS), as discussed by Menter et al. (2003) where the boundary layers separates at arbitrary locations based on the grid spacing. In order to avoid this, the DES concept has been extended to Delayed-DES (DDES), following the proposal of Menter et al. (2003) to ‘shield’ the boundary layer from the DES limiter (Shur et al. 2008). The dissipation term in the *k*-equation is then re-formulated as follows:

$$E_{DES} = \rho \frac{k^{3/2}}{\min\left(L_t; C_{DES}\Delta\right)} = \rho \frac{k^{3/2}}{L_t \min\left(1; C_{DES}\Delta / L_t\right)} = \rho \frac{k^{3/2}}{L_t} \max\left(1; \frac{L_t}{C_{DES}\Delta}\right)$$

$$E_{DDES} = \max\left(\frac{L_t}{C_{DES}\Delta} \cdot \left(1 - F_{DDES}\right), 1\right)$$

The function F_{DDES} is formulated in such a way as to give $F_{DDES}=1$ inside the wall boundary layer and $F_{DDES}=0$ away from the wall. The definition of this function is intricate as it involves a balance between save shielding and the desire to not suppress the formation of resolved turbulence as the flow leaves the wall.

2.3 Wall Modelled Large Eddy Simulation (WMLES)

The motivation for WMLES is to reduce the *Re* number scaling of wall-resolved LES. The principle idea is depicted in Figure 1. The near wall turbulence scales with the wall distance, *y*,, resulting in smaller and smaller eddies as the wall is approached. This effect is limited by viscosity, which damps out eddies inside the viscous sublayer (VS).

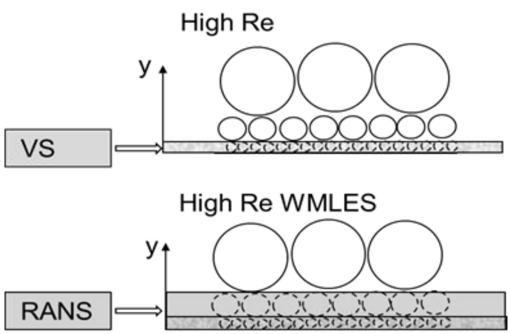

Fig. 1 Concept of WMLES for high Re number flows. Top: Wall-resolved LES. Bottom: WMLES.

As the *Re* number increases, smaller and smaller eddies appear, as the viscous sublayer becomes thinner. In order to avoid the resolution of these small near wall scales, RANS and LES models are combined in a way, where the RANS model covers the very near wall layer, and then switches over to an LES formulation once the grid spacing is sufficient to resolve the local scales. This is seen in Figure 1 bottom, where the RANS layer extends outside the VS and avoids the need to resolve the second row of eddies depicted in the sketch.

The WMLES formulation used in ANSYS-CFD is based on the formulation of Shur et al. (2008):

$$\nu_t = \min\left\{\left(\kappa y\right)^2, \left(C_{SMAG}\Delta\right)^2\right\}\left\{1 - \exp\left[-\left(y^+ / 25\right)^3\right]\right\} S$$

where again y is the wall distance, κ is the von Karman constant and S is the strain rate. This formulation was adapted to suit the needs of the ANSYS general purpose CFD codes. Near the wall, the min-function selects the Prandtl mixing length model whereas away from walls it switches over to the Smagorinsky (1963) model (with suitably defined cell size).

Figuer2 shows the results of a simulation of a boundary layer at Re_Θ=10000. Such a Re number is typically out of reach for wall-resolved LES due to the large grid resolution required. In the present study a grid with only ~$1.3 \cdot 10^6$ cells was used (Δx^+~700, Δz^+~350). Synthetic inlet turbulence was generated using the Vortex Method (Mathey et al. 2003).The logarithmic layer is captured very well as seen in Figure 2.

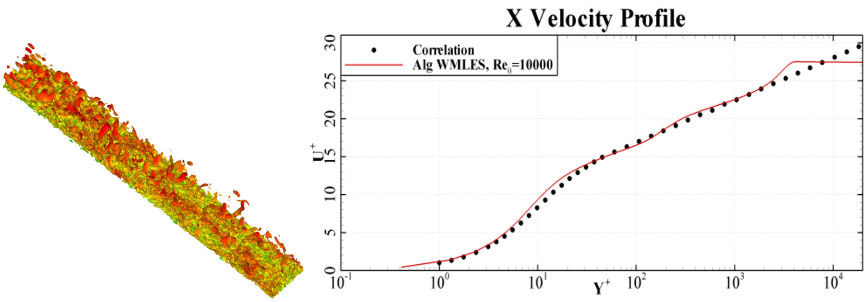

Fig. 2 Profile information for the flat plate boundary layer simulations. Re_Θ=10000.

2.4 Zonal/Embedded LES (ELES, ZLES)

The idea behind ZLES/ELES is to predefine different zones during the pre-processing stage with different treatment of turbulence (e.g. Cokljat et al. 2009, Menter et al. 2009). The domain is typically split into a RANS and a LES portion. Between these regions, the turbulence model is switched from RANS to LES (or WMLES). In order to maintain consistency, synthetic turbulence is generally introduced at RANS-LES interfaces. ELES is actually not a new model, but an infrastructure which combines existing elements of technology in a zonal fashion. The recommendations for each zone are therefore the same as given for the individual models.

2.5 Unsteady Inlet/Interface Turbulence

Classical LES requires providing unsteady fluctuations at turbulent inlets/interfaces (RANS-LES interface) to the LES domain. In the most general situation, the inlet profiles are not fully developed and no simple method exists of producing consistent inlet profiles. In such cases, it is desirable to generate synthetic turbulence based on given inlet profiles for RANS turbulence models. These inlet profiles are typically obtained from a pre-cursor RANS simulation of the domain upstream of the LES inlet. There are several methods for generating synthetic turbulence. In ANSYS-Fluent, the most widely used method is the Vortex Method (Mathey et al. 2003), where a number of discrete vortices are generated at the inlet/interface. Their distribution, strength and size are modeled to provide the desirable characteristics of real turbulence. The input parameters to the VM are the two scales from the upstream RANS simulation. An alternative to the VM is the generation of synthetic turbulence by using suitable harmonic functions used in ANSYS-CFX (Menter et al. 2009).

3 Flow Types and Modelling

3.1 Globally Unstable Flows

The classical example of globally unstable flows are flows past bluff bodies. Even when computed with a classical URANS model, will the simulation typically provide an unsteady output.

From a physical standpoint, such flows are characterized by the formation of 'new' turbulence downstream of the body, whereby this turbulence is independent from and effectively overrides the turbulence coming from the attached boundary layers around the body. In other words, the turbulence in the attached boundary layers has very little effect on the turbulence in the separated zone. The attached boundary layers can, however, define the separation point/line on a smoothly curved body and thereby affect the size of the separation zone.

Examples of globally unstable flows:

- Flows past bluff bodies
- Flows with strong swirl instabilities
- Flows with strong flow interaction

Of all flows where SRS modelling is required, globally unstable flows are the easiest to handle. They can typically be captured by a global RANS-LES model like SAS or DDES, without the need for generating synthetic turbulence at predefined interfaces or highly specialized grid generation procedures. Globally unstable flows are also the most beneficial for SRS, as experience shows that RANS models often fail on such flows with large margins of error. Fortunately, a large number of industrial flows fall into this category.

The safest SRS model for such flows is the SAS approach. It offers the advantage that the RANS model is not affected by the grid spacing and thereby avoids all potential negative effects of (D)DES, like 'grey zones' or grid induced separation. The SAS concept reverts back to (U)RANS in case the mesh/time step is not sufficient for LES and thereby preserves a strong 'backbone' of modelling independent of space and time resolution. SAS also avoids the need for shielding, which for internal flows with multiple walls can suppress turbulence formation in DDES models.

The alternative to SAS is DDES. If proper care is taken to ensure LES mesh quality in the detached flow regions, the model is operating in its design environment, typically providing high quality solutions.

In many cases, the behavior of SAS and DDES is very similar. The reason for recommending the SAS model lies in its safety due to the underlying RANS formulation.

Figure 3 shows the flow around a triangular cylinder in crossflow (Sjunnesson, 1992) as computed with the SST-SAS and the SST-DDES models. It is important to emphasize that the flow is computed with steady state boundary conditions (as would be employed for a RANS simulation). Still, the flow downstream of the obstacle turns quickly into unsteady (scale-resolving) mode, even though no unsteadiness is introduced by any boundary or interface condition.

The Reynolds number based on freestream velocity and edge length is Re=45,500 with an inlet velocity of 17.3 m/s. Periodic boundary conditions are applied in spanwise direction. The simulations where run with ANSYS-Fluent using the BCD (bounded central difference, see e.g. Jasak et al. 1999)) and CD (Central Difference) advection scheme and a time step of $\Delta t=10^{-5}s$ (CFL~1 behind cylinder).

The grid for the simulation around the triangular cylinder features 26 cells across its base. It is extended in the third direction to cover 6 times the edge length of the triangle with 81 cells in that direction. Due to the strong global instability of this flow, such a resolution is sufficient and has produced highly accurate solutions for mean flow and turbulence quantities. It should however be noted that not all flows feature such a strong instability as the triangular cylinder, and a higher grid resolution might then be required.

Figure 4 shows a comparison of results between SST-SAS, SST-DDES and experimental data. As can be seen, both models capture the flow well and agree with the experiments.

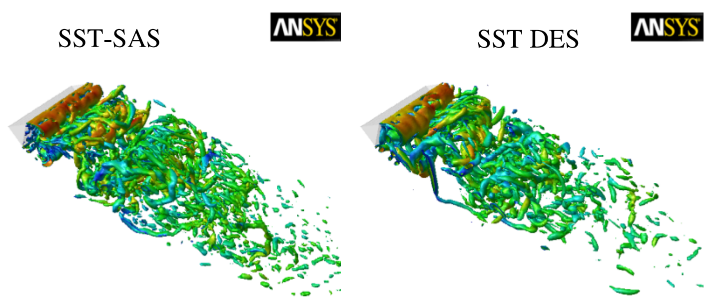

Fig. 3 Turbulence structures for flow around triangular cylinder in crossflow

3.2 Locally Unstable Flows

The expression 'locally' unstable flows is not easily definable as every turbulent flow is unstable by nature. Still in lieu of a more suitable expression, we mean here flows where a local shear layer generates an instability which turns the flow into a fully turbulent flow within a small number of shear layer thicknesses

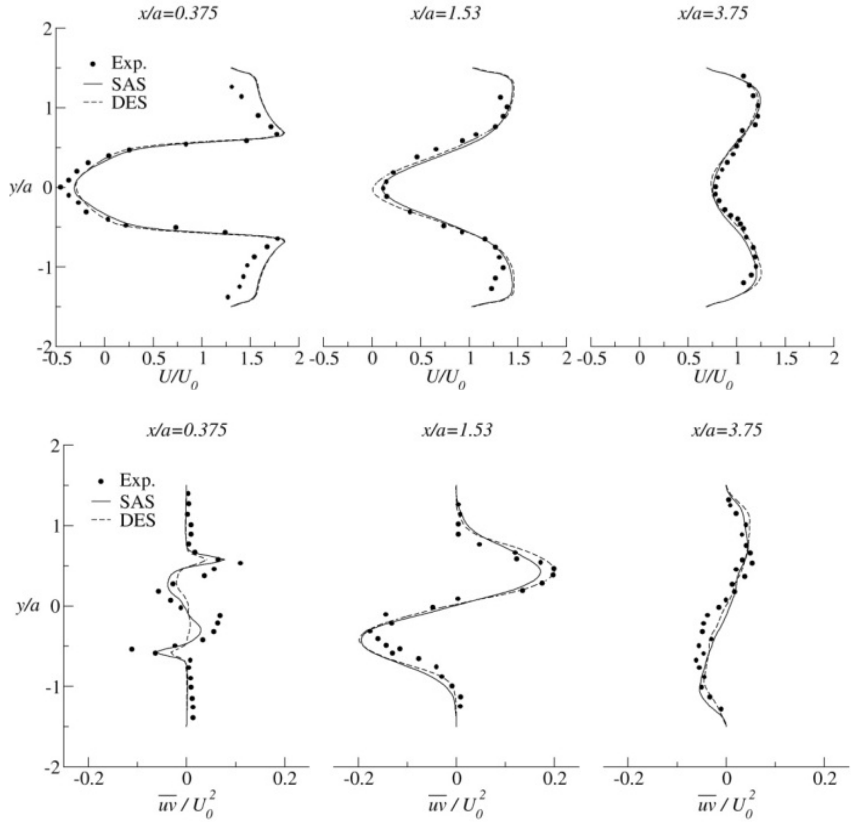

Fig. 4 Velocity profiles and turbulence RMS profiles for three different stations downstream of the triangular cylinder (x/a=0.375, x/a=1.53, x/a=3.75). Comparison of SST-SAS, SST-DES models and experiment. (a: U-velocity, b: u'v').

(<~3δ). To illustrate the rationale behind this definition, assume the computation of a mixing layer starting from two wall boundary layers (Figure 5). As the flat plate ends, the two boundary layers form a turbulent mixing layer, which becomes relatively quickly independent from the turbulence of the two boundary layers on the flat plate (yellow). The mixing layer instability (red) provides for a decoupling of the boundary layer and the mixing layer turbulence.

Examples of globally unstable flows:

- All equilibrium free shear flows (jets, wakes, mixing layers).
- Backward facing step flow
- Weakly interacting equilibrium flows
- Flows with weak swirl

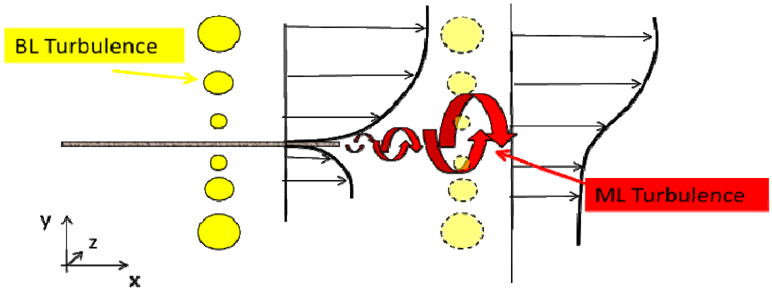

Fig. 5 Generic Mixing length example for locally unstable flows

The most general approach is the use of embedded or zonal RANS-LES methods, where the boundary layers are covered by a RANS model and the mixing layer by a LES model. The models will explicitly be switched from RANS to LES at a predefined interface. For a fully consistent simulation, one has to introduce synthetic turbulence at the RANS-LES interface.

A similar effect to simply switching the turbulence model at the interface can be achieved by the DDES model without an explicit interface between the RANS and the LES zones. The shielding function will ensure that the wall boundary layers are not affected by the LES part of the model and are covered in RANS mode. Slightly downstream of the trailing edge, the shielding function is deactivated and the model operates in LES mode if the grid and time step are of LES-quality. The model relies on the local instability of the mixing layer to produce the resolved turbulence content. The dashed yellow turbulence sketched in Figure 5 downstream of the trailing edge is neglected by this approach.

SAS models will typically remain in RANS or URANS mode for such flows. They should therefore not be used if unsteady flow characteristics are required. However, interfaces can be provided similar to zonal or embedded LES, where synthetic turbulence is introduced but the model is not switched. This then triggers the SAS model into SRS mode.

The recommendation for flows with local instabilities is to use ELES/ZLES models if the geometry and application allow the definition of well defined interfaces. One should introduce synthetic turbulence at these interfaces in order to preserve the balance between the RANS and LES turbulence content. In case the geometry/application is too complex and the definition of explicit RANS and LES zones is not easily possible, the DDES model should be applied.

The backward-facing step flow of Vogel and Eaton (1985) has been computed as an example using SST-DDES (see Gritskevich et al. (2011) for more details). In this flow, the Reynolds number based on a bulk velocity and on the step height H is equal to 28000, and the height of the channel upstream of the step is equal to 4H. The computational domain, see Figure 6, in the present study extended from -3.8H to 20H in streamwise direction (x=0 corresponds to the step location). In the spanwise direction, the size of the domain was 4H.

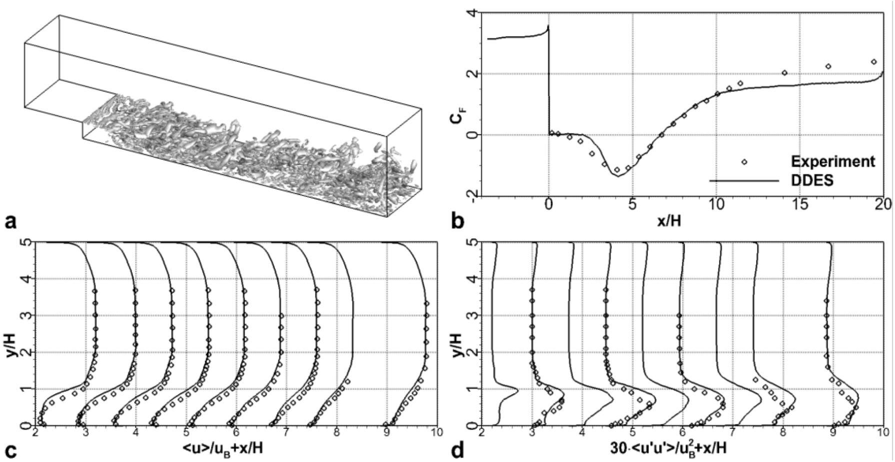

Fig. 6 A sketch of the flow (a) by SST-based DDES: b –skin friction coefficient distribution over the step-wall, c and d –profiles of streamwise velocity <u> and <u'u'> stress, e and f – iso-surfaces of Q criterion equal to 1 [s-2]. Profiles are plotted at x/H=2.2, 3.0, 3.7, 4.5, 5.2, 5.9, 6.7, 7.4, 8.7

The computational grid used in the simulation had 2.25 million hexahedral cells (2.3 million nodes) providing a near-wall resolution in wall units to be less than one. A non-dimensional time step was $\Delta t=0.02$ ensured the CFL number to be less than one in the entire domain. The number of cells in the spanwise direction was 80. At the inlet condition, steady state RANS profiles were imposed and unsteadiness results from the inherent flow instability past the step.

As seen in Figure 6, the skin friction distributions over the step-wall and velocity fields agree well with the data. This indicates that the turbulence from the upstream boundary layers (neglected in DDES past the step) is not essential for capturing the downstream flow development. The flow instability of the mixing layer is sufficiently quickly producing new turbulence to capture the main effects in the separation and recovery zone.

3.3 Stable Flows

3.3.1 Flow Physics

Stable flows in this context are characterized by a continuous development of the turbulence field. For such flows, the turbulence at a certain location depends strongly on the turbulence upstream of it. There is no mechanism of quickly generating 'new' turbulence and over-riding the upstream turbulence field. Stable flows in the context of this discussion are essentially wall-bounded flows - either attached or with small separation bubbles.

- Channel and pipe flows (attached and mildly separated)
- Boundary layers (attached and mildly separated)

For stable flows, the use of embedded or zonal RANS-LES methods with a well defined interface between the RANS and the LES zone is essential. Synthetic turbulence has to be introduced at the RANS-LES interface to ensure a proper balance between the modeled and the resolved content of turbulence. By such ‘injection’ of resolved turbulence, the balance between RANS and LES turbulence across the interface is preserved (assuming the synthetic turbulence is of sufficient quality). Neither DDES nor SAS-type models will be able to switch from RANS to SRS mode in such situations (e.g. Davidson 2006). Even an explicit switch from a RANS to an LES model (and the corresponding grid refinement) at the interface without an introduction of synthetic turbulence would not work well. If sufficient resolution is provided in the LES zone, the flow would eventually go through a transitional process and re-cover the fully turbulent state. However, such a process would require many boundary layer thicknesses with an entirely undefined model formulation in-between. This is in most technical flows not acceptable and has to be avoided.

The RANS-LES interface should be placed in a non-critical region of the flow (equilibrium flow), as the synthetic turbulence requires several boundary layer thicknesses to adjust and become ‘real’ turbulence.

As an alternative, the LES simulation can be carried out separately, on a reduced domain and by interpolating the ‘larger’ RANS solution onto the boundaries of the LES domain. At the inlet of such a domain, again synthetic turbulence needs to be generated.

The models selected in the RANS and LES zone depend on the flow physics. In the RANS zone, a suitable model for the flow should be selected. In the LES zone, the use of a WMLES formulation is typically recommended for wall boundary layers in order to avoid the unfavorable Reynolds number scaling of classical LES models. For free shear flows, the WALE (Nicoud and Ducros, 1999), model should provide good performance.

The following example is a flow through a pipe T-junction with two streams at different temperatures. This testcase was a used as a benchmark of the OECD to evaluate CFD capabilities for reactor safety applications. The geometry and grid are shown in Figure 7.

The grid consists of ~5 million hexahedral cells. This flow is not easily categorized into one of the three groups described above. In principle it can be computed with SAS and DDES models in SRS mode (not shown). This means that the instability in the interaction zone between the two streams is sufficiently strong to generate unsteady resolved turbulence. However, it was also observed, that these simulations are extremely sensitive to the details of the numerical method employed or the shielding function used. The SAS model provided ‘proper’ solutions only when a pure Central Difference scheme was selected, but went into URANS mode in case of the Bounded Central Difference scheme. The DDES model provided correct solutions, when a non-conservative shielding function is used but produces only weak unsteadiness in case of a conservative shielding function. It is therefore recommended to apply the ELES model, where modeled turbulence is converted into synthetic resolved turbulence in both pipes upstream of the interaction zone at

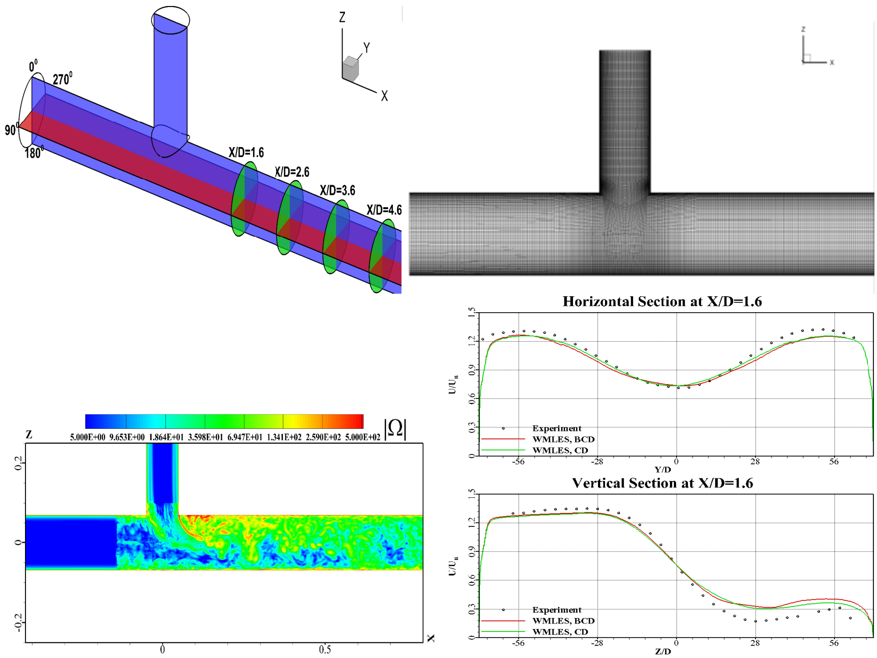

Fig. 7 T-Junction simulation. Upper left: geometry. Upper right: grid. Lower left: turbulence structures. Lower right: axial velocity at station X/D=1.6 for horizontal and vertical lines.

pre-defined RANS-LES interfaces. In addition, the turbulence model is switched from SST to WMLES at these interfaces. This then avoids the need for the flow instability of the interacting streams to generate resolved scales.

Figure 7 (lower left) shows that the resolved turbulence starts already upstream of the interaction zone due to the introduction of synthetic turbulence. Figure 7 (lower right) shows a comparison of computed and experimental axial velocity profiles in the main pipe at X/D=1.6. The method provides a good agreement between the simulations and the experimental data. It can also be seen that the switch from CD to BCD does not affect the solutions. This is different from the observation with the SAS model, which reacts sensitive to such changes in the current testcase.

4 Summary

An overview of hybrid RANS-LES models in industrial use has been provided. The main characteristics of the models have been described. It has been argued that there are three main types of flows, which require different strategies for hybrid modelling. Which model to select depends largely on the question, how strongly the resolved turbulence in the downstream ‘LES’ zone depends on the details of the turbulence in the upstream RANS zone. An attempt was made to define three different types of flows. In reality, there is clearly a substantial overlap between the flow types and a characterization is not always easy. However, the categories should help to conceptually understand which model to apply to which application.

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References

1. Cokljat, D., Caradi, D., Link, G., Lechner, R., Menter, F.R.: Embedded LES Methodology for General-Purpose CFD Solvers. In: Proc. 6th Int. Symp. Turbulence and Shear Flow Phenomena, Seoul, Korea, June 22-24, pp. 1191–1196 (2009)
2. Davidson, L.: Evaluation of the SST-SAS Model "Channel Flow, Asymmetric Diffuser and Axi-Symmetric Hill". In: Proceedings European Conference on Comp. Fluid Dyn. ECCOMAS CFD (2006)
3. Egorov, Y., Menter, F.R., Cokljat, D.: Scale-Adaptive Simulation Method for Unsteady Flow Predictions. Part 2: Application to Aerodynamic Flows. Journal Flow Turbulence and Combustion 85(1), 139–165 (2009)
4. Fröhlich, J., von Terzi, D.: Hybrid LES/RANS methods for simulation of turbulent flows. Progress in Aerospace Sciences 44(5), 349–377 (2008)
5. Gritskevich, M.S., Garbaruk, A.V., Menter, F.R.: Sensitization of DDES and IDDES Formulations to the k- ω Shear-Stress Transport Model. To be published: Journal Flow Turbulence and Combustion (2011)
6. Jasak, H., Weller, H.G., Gosman, A.D.: High Resolution Differencing Scheme for Arbitrarily Unstructured Meshes. Int. J. Numer. Meth. Fluids 31, 431–449 (1999)
7. Mathey, F., Cokljat, D., Bertoglio, J.P., Sergent, E.: Specification of LES Inlet Boundary Condition Using Vortex Method. In: 4th International Symposium on Turbulence, Heat and Mass Transfer, Antalya (2003)
8. Menter, F.R.: Two-equation eddy-viscosity turbulence models for engineering applications. AIAA Journal 32(8), 1598–1605 (1994)
9. Menter, F.R., Kuntz, M., Langtry, R.: Ten Years of Experience with the SST Turbulence Model. Turbulence, Heat and Mass Transfer 4, 625–632 (2003)
10. Menter, F.R., Garbaruk, A., Smirnov, P.: Scale adaptive simulation with artificial forcing. In: Proc. 3rd Symposium on Hybrid RANS-LES Methods (2009)
11. Menter, F.R., Egorov, Y.: Scale-Adaptive Simulation Method for Unsteady Flow Predictions. Part 1: Theory and Model Description. Journal Flow Turbulence and Combustion 85(1), 113–138 (2010)
12. Nicoud, F., Ducros, F.: Subgrid-scale stress modelling based on the square of the velocity gradient tensor. Flow, Turb. Combust. 62, 183–200 (1999)
13. Rotta, J.C.: Turbulente Strömumgen. BG Teubner Stuttgart (1972)
14. Sagaut, P., Deck, S., Terracol, M.: Multiscale and multiresolution approaches in turbulence. Imperial College Press, London (2006)
15. Shur, M.L., Spalart, P.R., Strelets, M.K., Travin, A.K.: A hybrid RANS-LES approach with delayed-DES and wall-modeled LES capabilities. International Journal of Heat and Fluid Flow 29, 1638–1649 (2008)

16. Sjunnesson, A., Henriksson, R., Lofstrom, C.: CARS measurements and Visualization of Reacting Flows in Bluff Body Stabilized Flame. AIAA Paper 92 – 3650 (1992)
17. Smagorinsky, J.: General Circulation Experiments with the Primitive Equations. Monthly Weather Review 91, 99–165 (1963)
18. Spalart, P.R., Jou, W., Strelets, M., Allmaras, S.: Comments on the feasibility of LES for wings, and on a hybrid RANS/LES approach. In: Advances in DNS/LES, 1st AFOSR Int. Conf. on DNS/LES (1997)
19. Spalart, P.R.: Strategies for turbulence modelling and simulations. Int. J. Heat Fluid Flow 21, 2 (2000)
20. Spalart, P., Deck, S., Shur, M., Squires, K., Strelets, M., Travin, A.: A New Version of Detached Eddy Simulation, Resistant to Ambiguous Grid Densities. Journal of Theoretical and Computational Fluid Dynamics 20, 181–195 (2006)
21. Strelets, M.: Detached Eddy Simulation of massively separated flows. AIAA Paper 2001-879 (2001)
22. Vogel, J.C., Eaton, J.K.: Combined heat transfer and fluid dynamic measurements downstream of a backward-facing step. Journal of Heat and Mass Transfer 107, 922–929 (1985)